Final Exam Review

1. Graph $f(x) = \begin{cases} 2x - 3, & x > 3 \\ 8 - x, & x \leq 3 \end{cases}$. Label at least four points. [Sec. 2.4, 2.5]
   Find the following limits.
   a) $\lim_{x \to 3^-} f(x)$ b) $\lim_{x \to 3^+} f(x)$ c) $\lim_{x \to 3} f(x)$

2. Graph $(x) = \begin{cases} x - 2, & x \geq 4 \\ 10 - 2x, & x < 4 \end{cases}$. Label at least four points. [Sec. 2.4, 2.5]
   Find the following limits.
   a) $\lim_{x \to 4^-} f(x)$ b) $\lim_{x \to 4^+} f(x)$ c) $\lim_{x \to 4} f(x)$

3. Find the limit, if it exists. If the limit is infinite, indicate whether it is $+\infty$ or $-\infty$. [Sec. 2.4]
   a) $\lim_{x \to -2} \frac{x^2+4x+4}{x^2+3x+2}$ b) $\lim_{x \to 7} \frac{x^2-8x+7}{x^2-6x-7}$ c) $\lim_{x \to 0^-} \frac{x(x^2-2)}{x^2}$
   d) $\lim_{x \to 16} \frac{\sqrt{x}-4}{x-16}$ e) $\lim_{x \to -\infty} \frac{x^2+x-5}{1-2x-x^3}$

4. The total cost, in dollars, to produce x units of a certain product is given by C(x) =22500 + 7.35x. The average cost is given by $C(\bar{x}) = \frac{22.500+7.35x}{x}$. Determine $\lim_{x \to \infty} C(\bar{x})$.Interpret it. [Sec. 2.4]

5. Use the definition of the derivative to find $f'(x)$. Show all the required steps. [Sec. 2.6]
   a) $f(x) = 4x^2 - 3x + 4$ b) $f(x) = 7 - x^2$

6. Find the derivative $f'(x)$. [Sec. 3.1]
   a) $f(x) = 5x^4 - 3x^2 + 6x - 3$ b) $f(x) = \sqrt[3]{x^2} - 2\sqrt{x}$
   c) $f(x) = \frac{2}{x^4} - \frac{3}{x^3} - 4x$ d) $f(x) = \frac{5}{\sqrt[3]{x^2}}$

7. Find the derivative $f'(x)$ using the product rule of derivatives. Simplify, if possible. [Sec. 3.2]
   a) $f(x) = (x^2 + x)(3x + 1)$ b) $f(x) = (5x^2 + 1)(2\sqrt{x} - 1)$

8. Find the derivative y' using the quotient rule of derivatives. Simplify, if possible. [Sec. 3.2]
   a) $y = \frac{x^4+x^2+1}{x^2+1}$ b) $y = \frac{x^2-x+1}{2x+5}$

9. Find the derivative $f'(x)$. Simplify, if possible. [Sec. 3.3]
   a) $f(x) = \frac{1}{(4x^3-4x+3)^3}$ b) $f(x) = \sqrt[3]{x^2 - 3x + 4}$

10. Evaluate the second derivative of the given function. [Sec. 3.5]
    a) $f''(1)$ for $f(x) = \frac{3x-2}{5x}$ b) $f''(1)$ for $f(x) = \sqrt{3x - 2}$

11. Solve the following applied problems. Interpret the results. [Sec. 3.4]
   a) A steel mill finds that its cost function is $C(x) = 8{,}000\sqrt{x} - 600\sqrt[3]{x}$ dollars, where x is the daily production of steel (in tons).
      i. Find the Marginal Cost Function.
      ii. Find the marginal cost when 64 tons of steel are produced.
      iii. Interpret the results.
   b) The manufacturer determines that the profit P (in dollars) derived from selling x units is given by $P(x) = 0.0004x^3 + 7x$.
      i. Find the Marginal Profit Function.
      ii. Find the marginal profit for a production level of 52 units.
      iii. Interpret the results.

12. A company's profit function is $P(x) = 13x - 1{,}700$. [Sec. 3.4]
   a) Find the Relative Rate of Change of the Profit Function.
   b) Find the Marginal Profit Function.
   c) Evaluate the Relative Rate of Change of the Profit Function at $x = 250$.

13. Elasticity of Demand and applications. Elasticity of Demand: $(p) = -\frac{pf'(p)}{f(p)}$. [Sec. 3.4]. A South American country exports coffee and estimates the demand function to be $f(p) = 63 - 2p^2$. If the country wants to raise revenue to improve the balance of payments, calculate its elasticity of demand and determine if it should raise or lower the prices from the present level of $3 per pound.

14. Solve the following applied problem. Interpret the results. [Sec. 3.5]
   The median age (in years) of the U.S. population over the decades from 1960 through 2010 is given by $f(t) = -0.2176t^3 + 1.962t^2 - 2.833t + 29.4$ for $(0 \le t \le 5)$, where t is measured in decades, with $t = 0$ corresponding to 1960.
   a) What was the median age of the population in the year 2000?
   b) At what rate was the median age of the population changing in the year 2000?

15. Consider the function $f(x) = -2x^3 + 3x^2 + 12x + 2$. [Sec. 4.3]
   a) Determine the intervals where f is increasing and decreasing.
   b) Find the relative extrema of f.
   c) Determine the concavity of the graph of f.
   d) Find the inflection points of f.

16. Find the absolute extreme values for each function on the given interval. [Sec. 4.4]
   a) $f(x) = x^3 - 2x^2 - 4x + 2$ on $[-1,3]$.
   b) $f(x) = x^3 + 6x^2 + 9x + 3$ on $[-4,2]$.

17. [Sec. 4.5]
   a) A company manufactures and sells x phones per week. The weekly price-demand and cost function are respectively $p(x) = 500 - 0.5x$ and $C(x) = 20{,}000 + 145x$
      What is the maximum weekly profit? How much should the company charge for the phones and how many phones should be produced to obtain the maximum weekly profit?

b) A deli sells 720 sandwiches per day at a price of \$6 each. A market survey shows that for every \$0.10 reduction in the price, 40 more sandwiches will be sold. What is the maximum daily revenue? How much should the deli charge for the sandwiches and how many sandwiches should be sold to obtain the maximum daily revenue?

18. Differentiation of Exponential Function. Find the derivative of the function. [Sec. 5.4]

a) $y = 6e^{3x^2}$ b) $y = x^2e^x$ c) $y = \ln(e^{4x} + 2)$

19. Differentiation of Natural Log. Find the derivative of the function. [Sec. 5.5]

a) $y = \ln x - \ln(x - 1)$ b) $y = \ln \frac{x^4}{x^2+9}$ c) $y = \ln \sqrt[3]{x^2 - 1}$

20. Find the integral. [Sec. 6.1]

a) $\int \frac{dx}{2\sqrt[3]{x^2}}$ b) $\int (x^4 - 9x^2 + 3)\, dx$ c) $\int \left(x^3 - 4 + \frac{5}{x^6}\right) dx$

21. Integration using logarithmic and exponential functions. [Sec. 6.2]

a) $8\int e^{-4x}\, dx$ b) $\int \left(e^{4x} - \frac{3}{e^{\frac{1}{2}x}}\right) dx$ c) $\int \frac{x-2x^3}{x^2}\, dx$

22. Find the particular solution $y = f(x)$ that satisfies the differential equation and initial condition. [Sec. 6.2]

a) Suppose that the marginal revenue from the sale of x units of the product is $MR = R' = 6e^{0.01x}$. At 0 units you have 0 revenue. What is the revenue in dollars from the sale of 100 units of the product?

b) The rate at which blood pressure decreases in the aorta of a normal adult after a heartbeat is $\frac{dp}{dt} = -46.645e^{-0.491t}$, where t is the time in seconds.

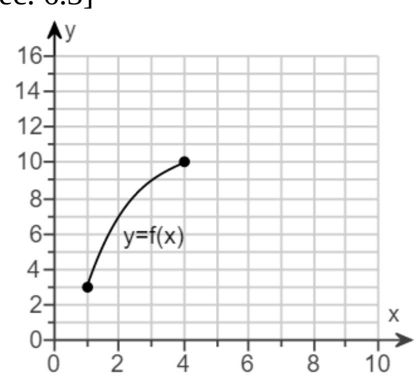
i. What function describes the blood pressure in the aorta if $p = 95$ when $= 0$?

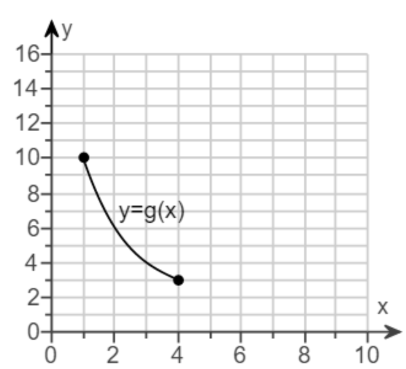
ii. What is the blood pressure 0.1 seconds after a heartbeat?

23. Evaluate the definite integral. [Sec. 6.3]

a) $\int_0^5 4\sqrt[3]{x^2}\, dx$ b) $\int_0^1 e^{3x}\, dx$ c) $\int_1^e \frac{4}{z}\, dz$

24. For each graph, estimate the area under the curve using Riemann sums for: i. L_3 ii. R_3 [Sec. 6.3]

a) 

b) 

25. Over the past decade, insurance companies have stepped up their advertising of bundling together the different types of insurance, such as life, health, and car, as a way for the consumer to realize savings through the bundling with one company. The rate of change in the number of life insurance companies per year in the United States that also sell accident and health insurance can be modeled by $f(x) = -63.07e^{-0.047x}$ where $1 \le x \le 7$and it represents the number of years since 2001 [Sec. 6.5].
   a) Evaluate $f(3)$ and interpret.
   b) Use the model to estimate the total increase or decrease in the number of life insurance companies that also sell accident and health insurance from 2004 to 2008.

26. Find the points of intersection and the bounded area between the two curves. [Sec. 6.6].
   a) $y = x^2$ and $y = x + 2$ b) $y = x - x^2$ and $y = x^2 - 4x$

27. The demand function for the sale of x bicycles is given by $d(x) = -0.3x + 330$ and the supply function is given by $S(x) = 0.2x + 75$. [Sec. 6.7]
   a) Graph the functions.
   Determine the:
   b) Equilibrium point.
   c) Consumers' surplus at the equilibrium point.
   d) Producers' surplus at the equilibrium point.

Key:

1. a) 5 b) 3 c) DNE
2. a) 2 b) 2 c) 2
3. a) 0 b) ¾ c) $+\infty$ d) 1/8 e) 0
4. 7.35. As the number of units produced increases, the average cost per unit approaches \$7.35.
5. a) $8x - 3$ b) $-2x$
6. a) $20x^3 - 6x + 6$ b) $\frac{2}{3\sqrt[3]{x}} - \frac{1}{\sqrt{x}}$ c) $-\frac{8}{x^5} + \frac{9}{x^4} - 4$ d) $-\frac{10}{3x\sqrt[3]{x^2}}$
7. a) $9x^2 + 8x + 1$ b) $\frac{5x^2+1}{\sqrt{x}} + 10x(2\sqrt{x} - 1)$
8. a) $\frac{2x^3(x^2+2)}{(x^2+1)^2}$ b) $\frac{2x^2+10x-7}{(2x+5)^2}$
9. a) $-\frac{3(12x^2-4)}{(4x^3-4x+3)^4}$ b) $\frac{2x-3}{3(x^2-3x+4)^{2/3}}$
10. a) -4/5 b) -9/4

11a. i) $C'(x) = \frac{4000}{\sqrt{x}} - \frac{200}{x^{2/3}}$ ii) 487.5
iii) The cost is increasing \$487.5 for every ton of steel when the mill is producing 64 tons of steel.

11b. i) $P'(x) = 0.0012x^2 + 7$ ii) 10.24
iii) The profit is increasing at \$10.24 per unit when the manufacturer produces 52 units.

12. a) $\frac{P'(x)}{P(x)} = \frac{13}{13x-700}$ b) $P'(x) = 13$ c) 0.008
13. $E(3) = \frac{4}{5}$; Raise price

14. a) $f(4) = 35.53$, Median Age (years) b) $f'(4) = 2.42$

15. a) Increasing: $(-1,2)$; Decreasing: $(-\infty,-1) \cup (2,\infty)$
b) Rel. Max: (2, 22) and Rel. Min: (-1, -5)
c) Concave up: $(-\infty,\frac{1}{2})$ and Concave down: $(\frac{1}{2},\infty)$
d) Inflection Point: $(\frac{1}{2}, 8.5)$

16. a) Min: (2, -6); Max: $(-\frac{2}{3}, 3.48)$ b) Min: (-4, -1) & (-1, -1); Max: (2,53)

17. a) The maximum profit is $43,012.50 by selling 355 phones at $322.50 per phone.
b) The maximum profit is $6,084 by selling 1560 sandwiches at $3.90 per sandwich.

18. a) $36xe^{3x^2}$ b) $xe^x(x+2)$ c) $\frac{4e^{4x}}{e^{4x}+2}$

19. a) $\frac{1}{x} - \frac{1}{x-1}$ b) $\frac{4}{x} - \frac{2x}{x^2+9}$ c) $\frac{2x}{3(x^2-1)}$

20. a) $\frac{3\sqrt[3]{x}}{2} + C$ b) $\frac{x^5}{5} - 3x^3 + 3x + C$ c) $\frac{x^4}{4} - 4x - \frac{1}{x^5} + C$

21. a) $-2e^{-4x} + C$ b) $\frac{e^{4x}}{4} + \frac{6}{e^{\frac{1}{2}x}} + C$ c) $\ln|x| - x^2 + C$

22. a) $1,030.97 b) i) $p = 95e^{-0.491t}$ ii) ≈ 90.45

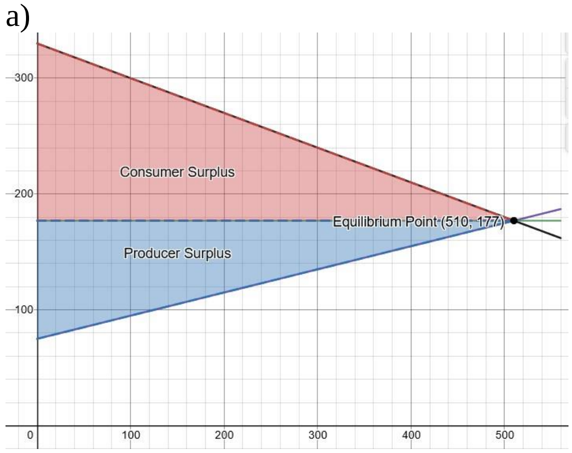
23. a) $(12)5^{2/3} \approx 35.088$ b) $\frac{e^3-1}{3} \approx 6.362$ c) 4

24. a) i) 19u ^{2} ii) 26u ^{2} b) i) 20u ^{2} ii) 13u ^{2}

25. a) In 2004, the number of life insurance companies in the United States that also sell accident and health insurance was decreasing at a rate of about 54.78 or 55 companies per year.
b) From 2004 to 2008, the total decrease in the number of life insurance companies in the United States that also sell accident and health insurance was about 199.74 or 200 companies.

26. a) $9/2$ b) $125/24 \approx 5.208$

27. a)



b) (510, $177) c) $39,015 d) $26,010