Math 1250G Final Exam Review

Updated Spring 2021

- ***Calculators are allowed on exam.***
- ***Exact, simplified solutions are required, unless otherwise stated.***
- ***All necessary work to arrive at a solution must be shown. If necessary steps are missing, no credit will be given for the problem.***
- ***When applicable, correct units must be given.***

1. Find the difference quotient, $\frac{f(x+h)-f(x)}{h}$ for the given functions.

   $a)\ f(x) = x^2 + 7x - 4$ $b)\ f(x) = \frac{2}{x+3}$

2. For the given functions: find $(f \circ g)(x)$, then state the domain of $(f \circ g)(x)$.

   $a)\ f(x) = \frac{5}{x+4}\ and\ g(x) = \frac{1}{x}$ $b)\ f(x) = \frac{x}{x+5}\ and\ g(x) = \frac{6}{x}$

3. Rewrite the following equations in standard form by completing the square and state the center and the radius. Graph each circle.

   $a)\ x^2 + y^2 + 6x + 2y + 6 = 0$ $b)\ x^2 + y^2 - 6y - 7 = 0$

4. Given $x = 4$ is one solution of $x^3 - 17x + 4 = 0$, algebraically determine all remaining solutions. Leave answers in exact, simplified form.

5. Find the complete factorization of $f(x) = 3x^3 - 10x^2 - x + 12$, given $x = \frac{4}{3}$ is a root of $f(x)$.

6. For the given rational functions, determine the following:

   i. Vertical, horizontal, and slant asymptotes.

   ii. x and y intercepts.

   iii. Graph the functions, label all asymptotes, intercepts, and at least 2 points on each piece of the graph.

   $a)\ f(x) = \frac{2x^2-5x+7}{x-2}$ b) $f(x) = \frac{x+3}{x^2+2x-8}$

7. Solve the inequalities algebraically by utilizing the following strategy. Write your answer using interval notation.

   i. Find the boundary points.
   ii. Using a number line, indicate with open/closed circles whether or not the boundary points will be included in the solution.
   iii. Use a test point in each interval to determine the solution.
   iv. Write the solution using interval notation.

   $a)\ x^2 - 5x + 4 < 0$ $b)\ 9x^2 + 3x - 2 \geq 0$

   $c)\ \frac{5x-1}{x+3} > 0$ $d)\ \frac{(x+4)(x-1)}{x+2} \leq 0$

8. The pitch of a musical tone varies inversely as its wavelength. A tone has a pitch of 660 vibrations per second and a wavelength of 1.6 feet. Determine an appropriate equation of variation and then determine the pitch of a tone that has a wavelength of 2.4 feet.

9. The number of minutes *M*, needed to solve an exercise set of variation problems varies directly as the number of problems *P*, and inversely as the number of people P_E, working to solve the problems. If it takes 4 people 32 minutes to solve 16 problems, determine the equation of variation and then use the equation to find how many minutes it will take 16 people to solve 24 problems.

10. Solve the following logarithmic and exponential equations. Write exact solutions.

$a)\ e^{4x-5} - 7 = 11{,}243$ $b)\ 7^{2x+1} = 3^{x+2}$

$c)\ log_3 y + 3log_3 y^2 = 14$ $d)\ log_4(2x+1) - log_4(x-3) = log_4(x+5)$

11. The population of Africa was 807 million in 2000, and grew to 1,052 million in 2011.Use the exponential growth model $A(t) = A_0 e^{kt}$ to find the exponential growth function that models this data. In what year will Africa's population reach 2 billion (2,000 million)?

12. The function $T(t) = 13.4\ln(t) - 11.6$ models the temperature increase, *T*, in an enclosed vehicle after *t* minutes when the outside air temperature is between 72°F and 92°F.

    a) Find the temperature increase, to the nearest degree, after 15 minutes.
    b) Find how long it takes, to the nearest minute, to reach a temperature increase of 35°F.

13. Without the use of a calculator, find the values of the six trigonometric functions at each angle.

a) $\frac{\pi}{6}$ b) $\frac{7\pi}{4}$ c) $\frac{2\pi}{3}$

14. Use the given information to find the exact value of each of the remaining six trigonometric functions.

a) $cot\theta = \frac{1}{3},\ \ \pi < \theta < \frac{3\pi}{2}$ b) $sin\theta = \frac{2}{5},\ \ \frac{\pi}{2} < \theta < \pi$

15. Find θ exactly, where $0 \le \theta < 2\pi$.

$a)\ sin\theta = -\frac{1}{2}$ $b)\ tan\theta = 1$

16. Find the amplitude, period, phase shift, vertical shift, and determine if the graph has a reflection. Then graph one period, beginning at the phase shift. Scale the graph using the five key points.

$a)\ y = 2\cos(x) + 1$ $b)\ y = \sin\left(2x - \frac{\pi}{3}\right) - 2$ $c)\ y = -3\cos(x + \pi)$

17. Use a sum or difference formula to find the exact value of the following:

$a)\ \tan\left(\frac{11\pi}{12}\right)$ $b)\ \sin 105°$

18. Use a half – angle formula to find the exact value of the following.

$a)\ \cos\left(\frac{11\pi}{12}\right)$ $b)\ \tan(15°)$

19. If $sin\theta = -\frac{4}{5}$, and $\pi < \theta < \frac{3\pi}{2}$, find

a) $\sin 2\theta$ $b)\ \cos\frac{\theta}{2}$

20. Solve the following triangles. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

$a)\ \mathrm{B} = 45°, \mathrm{A} = 60°, \mathrm{a} = \sqrt{6}$ $b)\ \mathrm{b} = 11, \mathrm{c} = 20,\ \mathrm{A} = 37°$

21. Solve each equation on the interval $[0,2\pi)$.

$a)\ \cos(2\theta) = \frac{1}{2}$ $b)\ \cos(2\theta) + \sin(\theta) = 0$ $c)\ 2sin^2\theta + cos\theta = 1$

22. Find the polar coordinates for each rectangular point.

$a)\ (-2,-2)$ $b)\ (7,0)$ $c)\ (0,-9)$

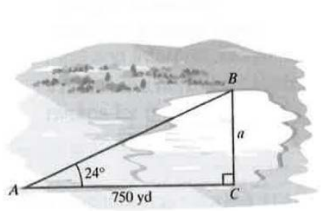
23. Find the rectangular points for each polar coordinate.

$a)\left(4,\frac{5\pi}{4}\right)$ $b)\ \left(\frac{1}{2},\frac{2\pi}{3}\right)$

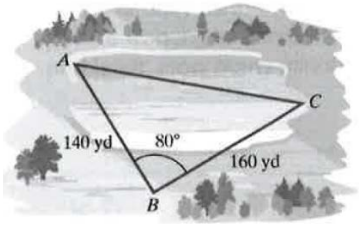
24. Convert the rectangular equations into a polar equation.

$a)\ x^2+y^2=16$ $b)\ 3x+8y=5$

25. The distance, a, across a lake is unknown. To find the distance, a surveyor took the measurements shown in the figure. What is the distance across the lake? Round to the nearest whole number.



26. Find the distance from A to C in the given picture. Round to the nearest yard.



27. Suppose you go for a hike on a bearing of S 31° W for 2.3 miles. Then you turn 90° clockwise and hike 3.5 miles on a bearing of N 59° W. At that time, what is your bearing from your start point? Round to the nearest degree.

28. Two fire lookout stations are 13 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N 35° E and the bearing of the fire from station B is N 49° W . How far to the nearest tenth of a mile, is the fire from station B?

Solutions

1.	a. $2x + h + 7$	b. $\dfrac{-2}{(x+3)(x+h+3)}$
2.	a. $(fog)(x) = \dfrac{6x}{1+4x}$ $\quad Domain: \left(-\infty, -\dfrac{1}{4}\right) \cup \left(-\dfrac{1}{4}, 0\right) \cup (0, \infty)$	
	b. $(fog)(x) = \dfrac{6}{6+5x}$ $\quad Domain: \left(-\infty, -\dfrac{6}{5}\right) \cup \left(-\dfrac{6}{5}, 0\right) \cup (0, \infty)$	
3.	a. $(x+3)^2 + (y+1)^2 = 4$ $\quad Center\ (-3,-1)$ $\quad radius = 2$	
	b. $x^2 + (y-3)^2 = 16$ $\quad Center\ (0,3)$ $\quad radius = 4$	
4.	$\{-2+\sqrt{5}, -2-\sqrt{5}\}$	
5.	$f(x) = (3x-4)(x-3)(x+1)$	
6.	a. Vertical Asymptote: $x = 2$, Horizontal Asymptote: None, Slant Asymptote: $y = 2x - 1$ Graph passes through: $(-3,-8), (0,-3.5), (1,-4), (3,10), (4,9.5), (7,14)$	
	b. Vertical Asymptotes: $x = -4, x = 2$, Horizontal Asymptote: $y = 0$, Slant Asymptote: None Graph passes through: $(-8,-.125), (-6,-.1875), (-3,0), (0,-.375), (6,.225), (12,\ .09375)$	
7.	a. $(1,4)$	b. $\left(-\infty, -\dfrac{2}{3}\right] \cup \left[\dfrac{1}{3}, \infty\right)$
	c. $(-\infty, -3) \cup \left(\dfrac{1}{5}, \infty\right)$	d. $(-\infty, -4] \cup (-2, 1]$
8.	Equation: $P = \frac{1056}{W}$. The pitch is 440 vibrations per second for a 2.4 foot wavelength.	
9.	Equation: $M = \frac{8P}{P_E}$. It will take 12 minutes for 16 people solve 24 problems.	
10.	a. $x = \dfrac{ln(11{,}250) + 5}{4}$	b. $x = \dfrac{2ln(3) - ln(7)}{2ln(7) - ln(3)}$
	c. $y = 9$	d. $x = 4$

11.	Equation: $A(t) = 807e^{0.0241t}$ Africa's population will reach 2 billion in 2038.	
12.	a. The temperature increase after 15 minutes is 25°F.	
	b. It will take 32 minutes to reach a temperature increase of 35°F.	
13.	a. $sin\frac{\pi}{6} = \frac{1}{2}, cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}, csc\frac{\pi}{6} = 2, sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}, cot\frac{\pi}{6} = \sqrt{3}$	
13.	b. $sin\ \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}, cos\ \frac{7\pi}{4} = \frac{\sqrt{2}}{2}, tan\ \frac{7\pi}{4} = -1, csc\ \frac{7\pi}{4} = -\sqrt{2}, sec\ \frac{7\pi}{4} = \sqrt{2}, cot\ \frac{7\pi}{4} = -1$	
	c. $sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}, cos\frac{2\pi}{3} = -\frac{1}{2}, tan\frac{2\pi}{3} = -\sqrt{3}, csc\frac{2\pi}{3} = \frac{2\sqrt{3}}{3}, sec\frac{2\pi}{3} = -2, cot\frac{2\pi}{3} = \frac{\sqrt{3}}{3}$	
14.	a. $cot\theta = \frac{1}{3}, tan\theta = 3, sin\theta = \frac{-3\sqrt{10}}{10}, cos\theta = \frac{-\sqrt{10}}{10}, csc\theta = \frac{-\sqrt{10}}{3}, sec\theta = -\sqrt{10}$	
	b. $sin\theta = \frac{2}{5}, cos\theta = \frac{-\sqrt{21}}{5}, tan\theta = \frac{-2\sqrt{21}}{21}, csc\theta = \frac{5}{2}, sec\theta = \frac{-5\sqrt{21}}{21}, cot\theta = \frac{-\sqrt{21}}{2}$	
15.	a. $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$	b. $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$
16.	a. Amplitude: 2 Period: 2π Phase Shift: None Key Points: $(0,3), \left(\frac{\pi}{2}, 1\right), (\pi, -1), \left(\frac{3\pi}{2}, 1\right) (2\pi, 3)$	Vertical Shift: Up 1 Reflection: None
	b. Amplitude: 1 Period: π Phase Shift: $\frac{\pi}{6}$ Key Points: $\left(\frac{\pi}{6}, -2\right), \left(\frac{5\pi}{12}, -1\right), \left(\frac{2\pi}{3}, -2\right), \left(\frac{11\pi}{12}, -3\right) (\frac{7\pi}{6}, -2)$	Vertical Shift: Down 2 Reflection: None
	c. Amplitude: 3 Period: 2π Phase Shift: $-\pi$ Key Points: $(-\pi, -3), \left(-\frac{\pi}{2}, 0\right), (0,3), \left(\frac{\pi}{2}, 0\right) (\pi, -3)$	Vertical Shift: None Reflection: Yes

17.	a. $\tan\left(\frac{11\pi}{12}\right) = \frac{1-\sqrt{3}}{1+\sqrt{3}} = -2+\sqrt{3}$	b. $\sin 105° = \frac{\sqrt{6}+\sqrt{2}}{4}$
18.	a. $\cos\left(\frac{11\pi}{12}\right) = -\frac{\sqrt{2+\sqrt{3}}}{2}$	b. $\tan(15°) = 2-\sqrt{3}$
19.	a. $\sin 2\theta = \frac{24}{25}$	b. $\cos\frac{\theta}{2} = -\frac{\sqrt{5}}{5}$
20.	a. $A = 60°, B = 45°, C = 75°$ $a = 2.4, b = 2, c = 2.7$	b. $a = 13.0, b = 11, c = 20$ $A = 37°, B = 31°, C = 112°$
21.	a. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$	b. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ c. $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$
22.	a. $\left(2\sqrt{2}, \frac{5\pi}{4}\right)$	b. $(7,0)$ c. $\left(9, \frac{3\pi}{2}\right)$
23.	a. $(-2\sqrt{2}, -2\sqrt{2})$	b. $\left(-\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$
24.	a. $r = 4$	b. $r = \frac{5}{3cos\theta + 8sin\theta}$
25.	334 *yards*	
26.	193 *yards*	
27.	*S* 87.7° W	
28.	10.7 miles	