Directions:

- A scientific calculator will be allowed on the final exam.
- Follow all directions including any rounding instructions.
- Unless the question asks for an estimate, give exact answers in completely reduced form.
- When appropriate, answers should include correct units.
- When specified, you must show work to receive credit for your answers.

1) The gross profit, *P*, for Exxon Mobil Corporation (in billions of dollars) *t* years since 2000 can be represented by the equation, $P = 12t + 13$. (Sections 1.2-1.3)

   a) Give the slope of this equation and write a sentence that explains its meaning in this situation.
   b) Give the *P*-intercept of this equation and write a sentence that explains its meaning in this situation.

2) The annual amount, *A*, of public expenditure on medical research (in billions of dollars) in the United States *t* years since 2000 can be estimated by the equation, $A = 4.02t + 25.8$. (Sections 1.2-1.3)

   a) Give the slope of this equation and write a sentence that explains its meaning in this situation.
   b) Give the *A*-intercept of this equation and write a sentence that explains its meaning in this situation.

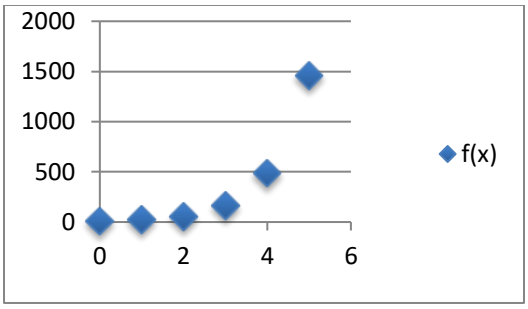
3) Let $f(x) = 2(x-3)^2 - 4$. Use this function to answer each question. You may sketch a graph to assist you. (Section 4.2)

   a) Does the graph of $f(x)$ open up or down? Explain how you know.
   b) What point is the vertex?
   c) What is the equation of the axis of symmetry?
   d) What point is the vertical intercept?
   e) What point is the symmetric point to the vertical intercept?
   f) State the domain and range for *f*(*x*).

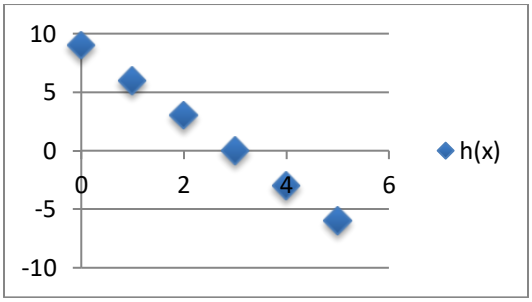
4) Let $f(x) = -\frac{1}{2}(x+2)^2 + 3$. Use this function to answer each question. You may sketch a graph to assist you. (Section 4.2).

   a) Does the graph of $f(x)$ open up or down? Explain how you know.
   b) What point is the vertex?
   c) What is the equation of the axis of symmetry?
   d) What point is the vertical intercept?
   e) What point is the symmetric point to the vertical intercept?
   f) State the domain and range for *f*(*x*).

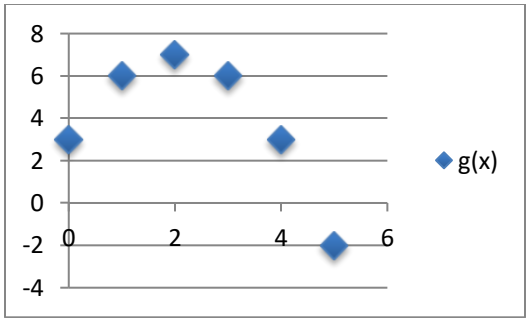
5) Tables of values and scatterplots for three functions are listed below. For each function, identify if it is linear, exponential or quadratic. If the function is linear, find the slope *m*. If the function is exponential, find the base multiplier *b*. If the function is quadratic, find the vertex. (Sections 1.6-1.7, 4.1, and 5.1)



x	*f(x)*
0	6
1	18
2	54
3	162
4	486
5	1458

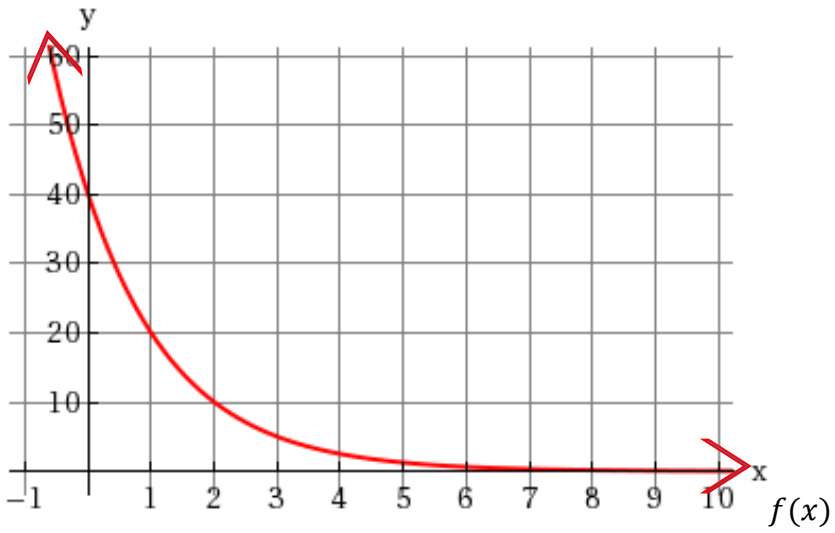


x	*h(x)*
0	9
1	6
2	3
3	0
4	-3
5	-6



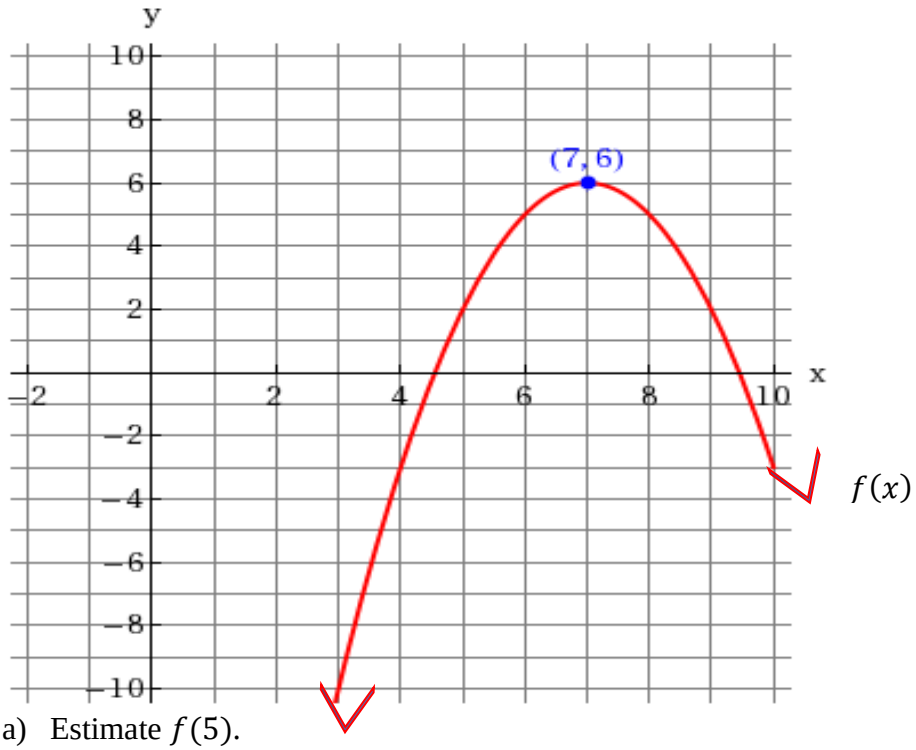
x	*g(x)*
0	3
1	6
2	7
3	6
4	3
5	-2

6) Use the graph of $f(x)$ below to answer each question. (Sections 5.1 and 5.3)



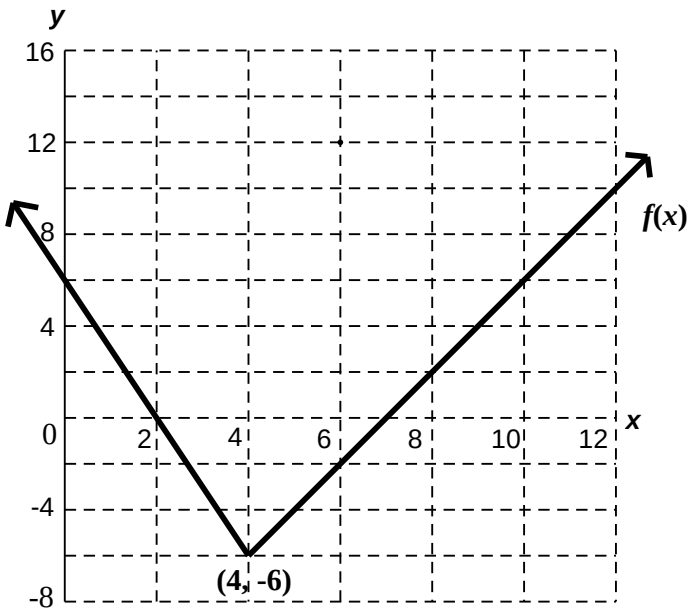
a) Estimate $f(1)$.
b) Estimate the x-value that makes $f(x) = 10$.
c) What point is the vertical intercept? Record answer as a point.
d) State the domain and range for $f(x)$.

7) Use the graph of $f(x)$ below to answer each question. (Sections 4.1-4.2)



a) Estimate $f(5)$.
b) Estimate the x-value(s) that makes $f(x) = 1$.
c) Estimate the horizontal intercept(s) for $f(x)$. Record answers as points.
d) State the domain and range for $f(x)$.

8) Use the graph of $f(x)$ below to answer each question. (Section 2.5)



a) Estimate $f(9)$.
b) Estimate the x-value(s) that make $f(x) = 6$.
c) Estimate the horizontal intercept(s) for $f(x)$. Record answers as points.
d) State the domain and range for $f(x)$.

9) Given $f(x) = 3^x$. (Sections 5.3, 6.1-6.3)
a) Find the inverse, $f^{-1}(x)$.
b) First complete the column of $f(x)$ values. Then use your knowledge of inverse functions to complete the last two columns.

x	$f(x)$		x	$f^{-1}(x)$
-1				
0				
1				
2				
3				

c) Sketch $f(x)$ and its inverse on the same set of axes. Include the line $y = x$. Label which graph is f and which graph is f^{-1}. Label any 6 points on the graph of f and f^{-1}. Give exact values for the coordinates.

10) Given $f(x) = 4^x$. (Sections 5.3, 6.1-6.3)

a) Find the inverse, $f^{-1}(x)$.

b) First complete the column of *f*(*x*) values. Then use your knowledge of inverse functions to complete the last two columns.

x	$f(x)$		x	$f^{-1}(x)$
-1				
0				
1				
2				
3				

c) Sketch $f(x)$ and its inverse on the same set of axes. Include the line $y = x$. Label which graph is *f* and which graph is f^{-1}. Label any 6 points on the graph of *f* and f^{-1}. Give exact values for the coordinates.

11) Let $f(x) = 16^x$. (Sections 1.5 and 3.1)

a) Find $f(0)$. b) Find $f(-1)$. c) Find $f(\frac{1}{2})$

12) Simplify the following and write all answers without negative exponents. (Section 3.1)

a) $\left(\frac{5a^5}{a^3b^4}\right)^{-1}$ b) $\left(\frac{18m^2n^{-2}}{9m^{-4}n^{-5}}\right)^{-3}$ c) $\left(\frac{25x^5y^{16}z^{-8}}{157a^4b^{-2}}\right)^{0}$

13) Let $g(x) = \log_6(x)$. (Section 1.5 and 6.2)

a) Find $g(36)$. b) Find $g(6)$. c) Find $g^{-1}(0)$. d) Find $g^{-1}(-1)$

14) Solve the following equations (Section 6.6)

a) $\log_4(x) = \frac{1}{2}$ b) $\log_2(x) = -1$ c) $\log(x) = 6$

15) Find the inverse of the following functions. (Sections 6.1-6.2)

a) $f(x) = \log x$ b) $g(x) = 6^x$ c) $h(x) = -3x + 1$

16) Write the following as a single logarithm with a coefficient of 1. (Section 6.4)

a) $3\log(x^4) - 2\log(3x)$

b) $2\log(2x^2) + 3\log(2x^6)$

c) $\ln(5x^3) + 2\ln(x) + 2\ln(3z) - 4\ln(yz)$

17) State the domain for each function below. (Section 7.1)

a) $f(x) = \frac{6x-12}{x^2-2x}$.

b) $f(x) = \frac{4x-16}{x^2-16}$.

18) Simplify the right side of the following function. (Section 7.2)

a) $f(x) = \frac{6x-12}{x^2-2x}$.

b) $f(x) = \frac{4x-16}{x^2-16}$.

19) Perform the indicated operation. Simplify the result. Show work. (Sections 7.3-7.4)

a) $\frac{y^2-6}{y^2+9y+18} - \frac{y-4}{y+6}$

b) $\frac{2x}{(x+2)(x-3)} + \frac{x}{x+2}$

c) $\frac{v+5}{v^2-6v-55} \div \frac{v-8}{v^2-4v-77}$

d) $\frac{2n+6}{n^2+8n+15} \cdot \frac{3n+15}{n^2+11n+28}$

20) Simplify. Assume that each variable is non-negative. Write the result in radical notation. (Section 8.1)

a) $\sqrt[3]{8m^3n^7}$ b) $\sqrt{24x^5y^{12}z}$ c) $\sqrt[3]{40a^6b^{10}}$

21) State the domain for each function below. (Section 8.2)

a) $f(x) = 2\sqrt{x+3} - 6$.

b) $f(x) = 8 + 2\sqrt{x-4}$.

22) Find x such that $f(x) = 0$ for each function below. (Section 8.5)

a) $f(x) = 2\sqrt{x+3} - 6$.

b) $f(x) = 8 + 2\sqrt{x-4}$.

23) Solve the absolute value equations below. (Section 2.5)

a) $|x+5| = 23$

b) $3|m-7| + 10 = 4$

c) $0 = 2|p-1| - 8$

24) In 2001, 408 teenagers underwent gastric bypass surgery. In 2003, 771 teenagers underwent gastric bypass surgery. (Section 1.4)

a) Use this information to write a linear model for the number of teenagers having gastric bypass surgery *t* years since 2000. Show work.
b) According to your model, what is the number of teenagers having gastric bypass surgery in 2006? Show work.

25) According to the Nutritional Sciences Department at Penn State University, a woman's optimal weight is 100 pounds at 5 feet and 130 pounds at 5 feet 6 inches. (Section 1.4)

a) Use this information to write a linear model for a woman's optimal weight in pounds based on the number of inches her height is above 5 feet. [Note: 1 foot=12 inches]
b) According to your model, what is the optimal weight of your math instructor if she is 5 feet 8 inches tall? Show work.

26) The equation, $D(t) = 650t + 8400$, represents the tuition, *D* dollars, at a college for the year that is *t* years since 2000. (Section 6.1)

a) Find an inverse function for this model. Show work.
b) Find the year when tuition was \$9,700. Show work.

27) The AEM Toy Company has a profit of $P(d) = 4.5d - 300$ dollars when *d* dolls are sold a month. (Section 6.1)

a) Find an inverse function for this model. Show work.
b) Find the amount of dolls needed to produce a profit of \$20,000. Show work.

28) An outlet electronics store has calculated from past sales data the revenue from selling refurbished iPads. The weekly revenue from selling these refurbished iPads can be modeled by $R(n) = -1.5(n - 60)^2 + 5700$ where $R(n)$ is the weekly revenue in dollars from selling n refurbished iPads. (Section 4.2)

a) What is the weekly revenue from selling 50 refurbished iPads?
b) Find the maximum weekly revenue and how many refurbished iPads must be sold per week to make the maximum revenue.

29) The total fresh vegetables consumed per person per year can be modeled by $V(t) = 0.7(t - 1.5)^2 + 95.4$ where $V(t)$ is the total fresh vegetables consumed per person in pounds per year t years since 2000. (Section 4.2)

a) Find $V(6)$ and explain its meaning. Round your answer to the nearest integer.
b) Find when the minimum intake of fresh vegetables per person in pounds occurred and what the minimum intake amount was.

30) Suppose the newest version of your favorite Smartphone comes out today. Assume that a total of 30 people go to your local cell phone dealer to purchase this phone today and that each day the total number of people who purchase the Smartphone doubles. Let $S(t)$ represent the number of people who have purchased the smartphone at *t* days since today. (Sections 5.1)

a) Find an equation for *S(t)*.
b) According to *S*(*t*), how many people have purchased the Smartphone 7 days from today?

31) Suppose a flu epidemic has broken out in all math 1215 courses at your school. Assume a total of 9 people have the flu as of today and that each day the total number of people who have the flu triples. Let *f*(*t*) represent the number of people who have the flu at *t* days since today. (Sections 5.1)

a) Find an equation for *f(t)*.
b) According to *f*(*t*), how many people at your school will have the flu 4 days from today?

32) Statistics indicate that the world population since World War II has been growing exponentially. If we assume exponential growth, the world population can be modeled by $P(t) = 4(1.019)^t$, where $P(t)$ is the world population in billions and *t* is the time in years since 1975. (Sections 5.5, 6.5)

a) Estimate the world population in 2015.
b) According to this model, when will the population reach 10 billion? Round to the nearest year. Show work.
c) According to this model, what was the world population in 1975?
d) According to this model, by what percent is the world population growing each year?

33) Suppose that $2000 is deposited into an account where the interest is compounded annually. This situation can be modeled by the function, $f(t) = 2000(1.052)^t$, where $f(t)$ represents the value (in dollars) of the account at *t* years after depositing the $2000. (Sections 5.5, 6.5)

a) In how many years will the money in the account double?
b) According to this model, how much money will the account have in 10 years?
c) According to this model, what is the earning interest rate?

34) The average forward speed for neotropical butterflies can be modeled by the function, $V(m) = 8.7\sqrt{m}$, where $V(m)$ represents the average forward body speed in meters per second and *m* represents the body mass of the butterfly in grams. (Sections 8.1 and 8.5)

a) Use this model to estimate the average forward speed of a butterfly with a body mass of 0.5 gram. Show work.
b) According to this model, what would be the body mass of a butterfly that has an average forward speed of 5 meters per second? Round to one decimal place.

35) The approximation of the body length of mammals based on the mammal's body mass can be expressed through $L(M) = 0.330\sqrt[3]{M}$, where *L(M)* represents the body length in meters of a mammal with a body mass of *M* kilograms. (Sections 8.1 and 8.5)

a) Use this model to estimate the body length of a mammal with a body mass of 4.6 kilograms. Show work.
b) With this model, estimate the body mass of a mammal with a body length of 1 meter. Round to one decimal place.

Solutions

1) a) The slope is 12. Exxon Mobile's profit increases by 12 billion dollars yearly.
b) The P-intercept is (0, 13). Exxon Mobile Corporation had 13 billion dollars in profit in 2000.
2) a) The slope is 4.02. The public expenditure on medical research increases 4.02 billion dollars yearly.
b) The A-intercept is (0, 25.8). The public expenditure on medical research was 25.8 billion dollars in the year 2000.
3) a) The graph opens up because a is positive.
b) The vertex is $(3, -4)$.
c) The axis of symmetry is $x = 3$.
d) The vertical intercept is (0, 14)
e) The symmetrical point to the vertical intercept (6, 14)
f) The domain is $(-\infty, \infty)$ and the range is $[-4, \infty)$.
4) a) The graph opens down because a is negative.
b) The vertex is $(-2, 3)$.
c) The axis of symmetry is $x = -2$.
d) The vertical intercept is (0, 1)
e) The symmetrical point to the vertical intercept $(-4, 1)$
f) The domain is $(-\infty, \infty)$ and the range is $(-\infty, 3]$.
5) a) f is exponential, b =3; h is linear, m = -3; g is quadratic, vertex (2,7)
6) a) $f(1) = 20$; b) $f(x) = 10\ when\ x = 2$; c) Vertical intercept (0, 40); d) Domain$(-\infty, \infty)$ and range $(0, \infty)$.
7) a) $f(5) = 2$; b) $f(x) = 1\ when\ x \approx 4.8\ and\ 9.2$; c) horizontal intercepts ≈(4.5, 0) and (9.5, 0); d) Domain$(-\infty, \infty)$ and range $(-\infty, 6]$.
8) a) $f(9) \approx 4$; b) $f(x) = 6\ when\ x \approx 0\ and\ 10$; c) horizontal intercepts ≈(2, 0) and (7, 0); d) Domain$(-\infty, \infty)$ and range $[-6, \infty)$.
9) a) $f^{-1}(x) = \log_3(x)$; b) sample points for f: $\left(-1, \frac{1}{3}\right), (0, 1), (2, 9)$, $and\ f^{-1}$sample points $\left(\frac{1}{3}, -1\right), (1, 0), (9, 2)$; c) graph for f in Quads I and II, and f^{-1} in Quads I and IV.
10) a) $f^{-1}(x) = \log_4(x)$; b) sample points for f: $\left(-1, \frac{1}{4}\right), (0, 1), (2, 16)\ and\ f^{-1}$ sample points $\left(\frac{1}{4}, -1\right), (1, 0), (16, 2)$; c) graph for f in Quads I and II, and f^{-1} in Quads I and IV.
11) a)$f(0) = 1$; b) $f(-1) = \frac{1}{16}$; c) $f\left(\frac{1}{2}\right) = 4$.
12) a)$\frac{b^4}{5a^2}$; b) $\frac{1}{8m^{18}n^9}$; c) 1
13) a) g(36)=2; b) g(6) =1; c) g^{-1}(0)=1; d) g^{-1}(-1)= $\frac{1}{6}$
14) a) $x = 2$; b) $x = \frac{1}{2}$; c) $x = 1{,}000{,}000$
15) a) $f^{-1}(x) = 10^x$; b) $g^{-1}(x) = \log_6(x)$; c) $h^{-1}(x) = \frac{1-x}{3}$
16) a) $\log\left(\frac{x^{10}}{9}\right)$; b) $\log(32x^{22})$; c)$\ln\left(\frac{45x^5}{y^4z^2}\right)$
17) a) All real numbers except 0 and 2. b) All real numbers except 4 $and\ -4$.
18) a) $f(x) = \frac{6}{x}$ b) $f(x) = \frac{4}{x+4}$

19) a) $\frac{1}{y+3}$; b) $\frac{x(x-1)}{(x+2)(x-3)}$; c) $\frac{v+7}{v-8}$; d) $\frac{6}{(n+4)(n+7)}$
20) a) $2mn^2\sqrt[3]{n}$; b) $2x^2y^6\sqrt{6xz}$; c) $2a^2b^3\sqrt[3]{5b}$
21) a) $x \geq -3$; b) $x \geq 4$
22) a) $x = 6$; b) no solution ($x = 20$ is extraneous)
23) a) $x = 18\ or\ x = -28$; b) no solution; c) $p = -3\ or\ p = 5$

24) a)$G(t) = 181.5t + 226.5$; b) In 2006, 1,316 teenagers had gastric bypass surgery.

25) a) $W(h) = 5h + 100$; b) According to the model, the optimal weight for my math instructor is 140 pounds.

26) a) $t(D) = \frac{D-8400}{650}$; b) In 2002, the tuition was \$9,700.

27) a) $d(P) = \frac{P+300}{4.5}$; b) According to the model, 4,512 dolls are needed to produce a profit of \$20,000.

28) a) $R(50) = 5550\ dollars$; b) Maximum (60, 5700). The weekly revenue from selling refurbished iPads reaches a maximum of 5700 dollars for selling 60 iPads.

29) a) $V(6) = 110\ pounds$ meaning In 2006, the total fresh vegetables consumed per person was about 110 pounds; b) Minimum (1.5, 95.4). Midway through 2001, the total amount of fresh vegetables consumed per person reached a minimum of about 95.4 pounds.

30) a) $S(t) = 30(2)^t$; b) According to the model, 3,840 people would have purchased the Smartphone in 7 days.

31) a) $f(t) = 9(3)^t$; b) According to the model, 729 people will have the flu in 4 days.

32) a) According to the model, the world population will be 8.5 billion in 2015.
b) The world population will reach 10 billion in 2024.
c) According to this model, the world population was 4 billion in 1975.
d) The world population is growing each year by 1.9%.

33) a) According to this model, the account will have doubled in 14 years.
b) In 10 years, the account will have \$3, 320.38.
c) According to this model, the earning interest rate is 5.2%.

34) a) A butterfly with the body mass of 0.5 grams would have a forward speed of approximately 6.2 meters per second.
b) According to this model, a butterfly with the body mass of 0.3 will have an airspeed of 5 meters per second.

35) a) According to the model, the approximate length of a mammal with a body mass of 4.6 is 0.5 meters.
b) The body mass of a mammal with the body length of 1 meter is 27.8 kilograms.