Directions:

- Unless the question asks for an estimate, give an exact answer (real or complex) in completely reduced form.
- When appropriate, answers should include correct units.
- When specified, you must show work to receive credit for your answers.
- A scientific calculator may be used on the final exam.

Formulas:

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, $A = Pe^{rt}$, $A = P\left(1+\frac{r}{n}\right)^{nt}$

1) Find the difference quotient of f, $\frac{f(x+h)-f(x)}{h}$ $where\ h \neq 0$, and simplify. [1.4]
   a) $f(x) = 7 - 12x$
   b) $f(x) = x^2 + 1$
   c) $f(x) = x^2 - 2x$
   d) $f(x) = 2x^2 + 3x - 6$

2) Let $g(x) = 5(x-23)^2 - 20$. [3.1, 3.2, 3.5, 4.4]
   a) Give the coordinates of the vertex.
   b) Explain how $g(x)$ is transformed from the graph of $y = x^2$.
   c) What are the zeros of $g(x)$?

3) Let $g(x) = -\frac{1}{2}(x+5)^2 + 8$. [3.1, 3.2, 3.5, 4.4]
   a) Give the coordinates of the vertex.
   b) Explain how $g(x)$ is transformed from the graph of $y = x^2$.
   c) What are the zeros of $g(x)$?

4) Write the new function $f(x)$ that satisfies the following conditions: $y = |x|$ is reflected with respect to the x-axis, compressed by a factor of $\frac{1}{3}$, shifted to the left three units, and up five units. [3.5]

5) Write the new function $f(x)$ that satisfies the following conditions: $y = x^5$ is stretched by a factor of 3, shifted to the right three units, and down five units. [3.5]

6) Solve using the most appropriate algebraic method. Show work. [3.2, 3.3]
   a) $4x^2 + 2x + 1 = 0$
   b) $-2x^2 + 4x - 8 = 0$
   c) $7x^2 = -28$
   d) $-24 = x^2 - 11x$
   e) $25 = x^2 - 8x$

7) The cost function for a product is given by $C(n) = n^2 + 12n + 2100$, where n is the number of units produced and sold and $C(n)$ is the cost in dollars. [3.2, 5.2]
   a) What is the domain of this function in context of the application?
   b) Is the function one-to-one for the domain in part (a)? Explain.
   c) Find the inverse of this function for the domain in part (a). Show work. (Hint: Use completing the square).
   d) Use the inverse function in part (c) to find how many units are produced and sold if the cost is $3,220. Show work.

8) The cost function for a product is given by $C(n) = n^2 + 6n + 640$, where n is the number of units produced and sold and $C(n)$ is the cost in dollars. [3.2, 5.2]

a) What is the domain of this function in context of the application?
b) Is the function one-to-one for the domain in part (a)? Explain.
c) Find the inverse of this function for the domain in part (a). Show work. (Hint: Use completing the square).
d) Use the inverse function in part (c) to find how many units are produced and sold if the cost is $2,312. Show work.

9) Graph by hand. [2.4, 4.2]:
a) $f(x)\begin{cases} x^2-1, & x<-2 \\ 3x+2, & -2\le x<2 \\ \sqrt{x+2}, & x\ge 2 \end{cases}$

b) $f(x)=\begin{cases} x-2, & x\le 1 \\ \sqrt{3x+1}, & x>1 \end{cases}$

c) $f(x)=\begin{cases} x^2-1, & x<1 \\ 1-2x, & x\ge 1 \end{cases}$

10) Let $f(x)=3x-4$ *and* $g(x)=\frac{x+4}{3}$. [5.1, 5.2]
a) Find $f(g(x))$.
b) Find $g(f(x))$.
c) Are $f(x)$ *and* $g(x)$ inverse functions? Explain.

11) Let $f(x)=-8x$ *and* $g(x)=\frac{1}{8}x$. [5.1, 5.2]
a) Find $f(g(x))$.
b) Find $g(f(x))$.
c) Are $f(x)$ *and* $g(x)$ inverse functions? Explain.

12) Let $f(x)=2x+6$ *and* $g(x)=\frac{x-6}{2}$. [5.1, 5.2]
a) Find $f(g(x))$.
b) Find $g(f(x))$.
c) Are $f(x)$ *and* $g(x)$ inverse functions? Explain.

13) Find the inverse of each function. [5.2]
a) $w(x)=2x^3-5$
b) $f(x)=\sqrt[5]{x+7}$

14) Solve each of the following equations algebraically. [2.5, 4.8]
a) $\sqrt{7x-28}=\sqrt{x^2-4x}$
b) $|x-18|=x^2-18x$
c) $|x^2+2x-4|=4$

15) For each function listed below, identify the type of function and then give the domain and range using interval notation. [1.3, 2.5, 5.4]
a) $g(x)=3x^2-20$
b) $h(x)=3\ln(x)$
c) $k(x)=|2x+3|-8$
d) $j(x)=3(4^x)$

16) At the end of an advertising campaign, the weekly sales declined. The weekly sales, y (in dollars), are modeled by the equation $y = 12{,}000(2^{-0.08x})$, where x is the number of weeks after the end of the campaign. [5.3]
   a) Determine the sales at the end of the campaign.
   b) Determine the sales 6 weeks after the end of the campaign.
   c) Does the model indicate that sales eventually reach $0? Explain.

17) At the end of an advertising campaign, the weekly sales declined. The weekly sales, y (in dollars), are modeled by the equation $y = 9{,}000(3^{-0.06x})$, where x is the number of weeks after the end of the campaign. [5.3]
   a) Determine the sales at the end of the campaign.
   b) Determine the sales 9 weeks after the end of the campaign.
   c) Does the model indicate that sales eventually reach $0? Explain.

18) Solve each equation algebraically. Show work. When necessary, round answers to four decimal places. [5.6]
   a) $\log_4(x) = -2$
   b) $4 + \log(x) = 10$
   c) $e^{(-2x+3)} = 2$
   d) $300 = 1200(2^{-0.1x})$
   e) $2^{5x-9} = 35$
   f) $\ln(-2x + 3) = 10$
   g) $2\ln(x) + 7 = \ln(4x) + 10$

19) Rewrite as a single logarithm. [5.5]
   a) $2\log(x) + 5\log(y) - 8\log(z)$
   b) $3\log_2(m) - 2\log_2(n) + \log_2(q)$
   c) $4\ln(x) - 7\ln(y) + 3\ln(z) - \ln(x)$

20) Rewrite as the sum, difference, or product of logarithms and simplify if possible. [5.5]
   a) $ln\left(\frac{y^2e^{3x}}{z^3}\right)$
   b) $log\left(\frac{a^3c^5}{b^7}\right)$

21) Suppose $ 9,000 is invested is invested for t years at 5.5% interest compounded monthly. [5.3]
   a) Write an equation that gives the future value, S.
   b) Using the model in (a), find the future value of the investment in 4 years.
   c) Using the model in (a), find the number of years it will take the investment to double.

22) Suppose $ 6,000 is invested is invested for t years at 7.2% interest compounded quarterly. [5.3]
   a) Write an equation that gives the future value, S.
   b) Using the model in (a), find the future value of the investment in 5 years.
   c) Using the model in (a), find the number of years it will take the investment to triple.

23) Let $f(x) = 3x^3 + 18x^2 - 12x - 72$. Use this function to answer each question. [4.1, 4.2, 4.4]
   a) State the degree and leading coefficient of $f(x)$.
   b) Find all x such that $f(x) = 0$. Solve algebraically. Show work
   c) Describe the end behavior of the graph of $f(x)$.
   d) How many turning points does the graph of $f(x)$ have?

24) Let $f(x) = -x^3 + 4x^2 + 9x - 36$. Use this function to answer each question. [4.1, 4.2, 4.4]

a) State the degree and leading coefficient of $f(x)$.
b) Find all x such that $f(x) = 0$. Solve algebraically. Show work
c) Describe the end behavior of the graph of $f(x)$.
d) How many turning points does the graph of $f(x)$ have?

25) Solve algebraically. Show work. [4.4]

a) $0 = 4x^3 - 4x$
b) $x^3 - 15x^2 + 56x = 0$
c) $0 = 2x^4 - 3x^3 - 20x^2$
d) $x^4 - 3x^3 + 2x^2 = 0$

26) Find the domain, vertical and horizontal asymptotes for each of the following. [4.6]

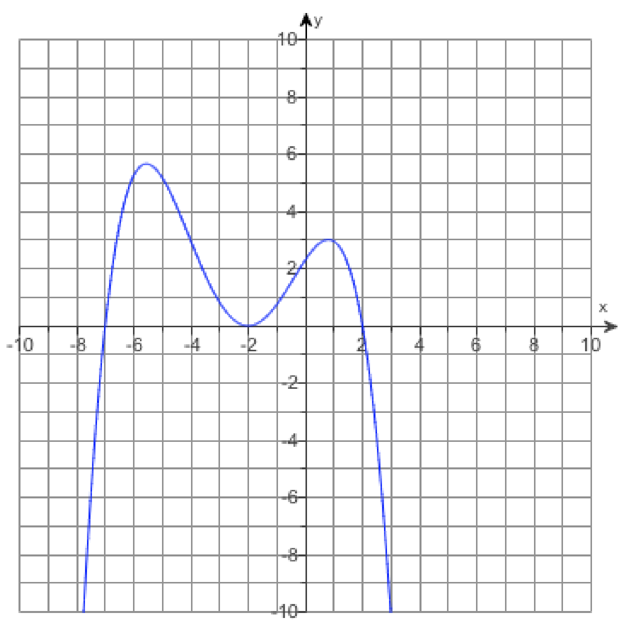
a) $f(x) = \frac{1}{(x-1)^2} + 1$

b) $g(x) = \frac{1-5x}{2x+1}$

c) $f(x) = \frac{x+5}{x^2+7x+10}$

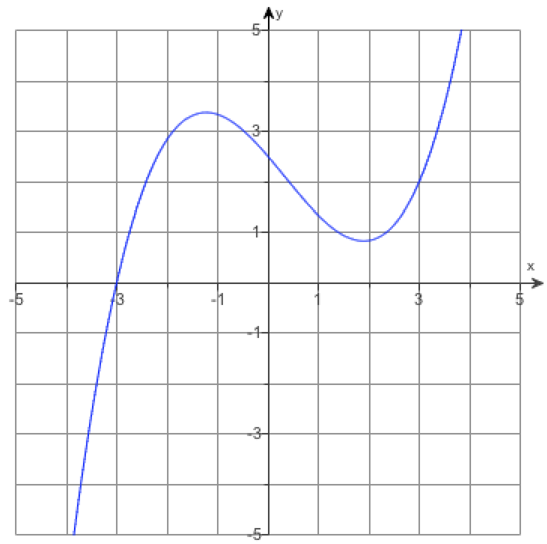
27) Use the graph of the polynomial function f to complete the following. Let a be the leading coefficient of the polynomial $f(x)$. [4.2]

a) Determine the number of turning points.
b) Estimate the x-intercepts.
c) State whether $a > 0$ *or* $a < 0$.
d) Determine the minimum degree of f.



28) Use the graph of the polynomial function f to complete the following. Let a be the leading coefficient of the polynomial $f(x)$. [4.2]

a) Determine the number of turning points.
b) Estimate the x-intercepts.
c) State whether $a > 0$ or $a < 0$.
d) Determine the minimum degree of f.



Solutions:

1) a) -12
   b) $2x + h$
   c) $2x + h - 2$

2) a) $(23, -20)$
   b) Stretched by a factor of 5, shift right 23 units, and shift down 20 units.
   c) $x = 21, x = 25$

3) a) $(-5, 8)$
   b) Reflection with respect to the x-axis, compressed by a factor of $\frac{1}{2}$, left 5 units, and shift up 8 units.
   c) $x = -9, x = -1$

4) $f(x) = -\frac{1}{3}|x + 3| + 5$

5) $f(x) = 3(x - 3)^5 - 5$

6) a) $x = \frac{-1 \pm i\sqrt{3}}{4}$
   b) $x = 1 \pm i\sqrt{3}$
   c) $x = \pm 2i$
   d) $x = 8, 3$
   e) $x = 4 \pm \sqrt{41}$

7) a) $n \geq 0\ or\ [0, \infty)$
   b) Yes. Possible answers for explanation: the domain restrictions allows the function to pass the horizontal line test.
   c) $n(C) = -6 + \sqrt{C - 2064}$
   d) 28 units

8) a) $n \geq 0\ or\ [0, \infty)$
   b) Yes. Possible answers for explanation: the domain restrictions allows the function to pass the horizontal line test.
   c) $n(C) = -3 + \sqrt{C - 631}$
   d) 38 units

9) a) For $x < -2$: decreasing concave up with an open circle at $(-2,3)$; For $-2 \leq x < 2$: line segment with closed circle at $(-2, -4)$ and open circle at $(2,8)$; For $x \geq 2$: Increasing, concave down with a closed circle at $(2, 2)$
   b) For $x \leq 1$: increasing line with closed circle at $(1, -1)$; For $x > 1$: increasing, concave down with open circle open circle at $(1, 2)$
   c)For $x < 1$: Increasing, concave up with open circle at $(1, 0)$; For $x \geq 1$: decreasing line with closed circle at $(1, -1)$

10) a) $f\left(g(x)\right) = x$
b) $g\left(f(x)\right) = x$
c) Yes; give explanation

11) a) $f\left(g(x)\right) = -x$
b) $g\left(f(x)\right) = -x$
c) No; give explanation

12) a) $f\left(g(x)\right) = x$
b) $g\left(f(x)\right) = x$
c) Yes; give explanation

13) a) $w^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$
b)) $f^{-1}(x) = x^5 - 7$

14) a) $x = 4 \; and \; x = 7$
b) $x = -1 \; and \; x = 18$
c) $x = \; -4, 2, -2, 0$

15) a) $g(x)$: quadratic, domain : $(-\infty, \infty)$, range: $[-20, \infty)$
b) $h(x)$: logarithmic, domain : $(0, \infty)$, range: $(-\infty, \infty)$
c) $k(x)$: absolute value, domain : $(-\infty, \infty)$, range: $[-8, \infty)$
d) $g(x)$: exponential, domain : $(-\infty, \infty)$, range: $(0, \infty)$

16) a) 12,000 dollars
b) $\approx$ 8604 dollars
c) No; the model has a horizontal asymptote at y=0

17) a) 9,000 dollars
b) $\approx$ 4973 dollars
c) No; the model has a horizontal asymptote at y=0

18) a) $x = \frac{1}{16}$
b) $x = 1{,}000{,}000$
c) $x \approx 1.1534$
d) $x = 20$
e) $x = 3$

f) $x \approx -11011.7329$
g) $x \approx 80.3421$

19) a) $\log\left(\frac{x^2y^5}{z^8}\right)$
b) $\log_2\left(\frac{m^3q}{n^2}\right)$

20) a) $2\ln(y) + 3x - 3\ln(z)$
b) $3\log(a) - 7\log(b) + 5\log(c)$

21) a) $S = 9000\left(1 + \frac{0.055}{12}\right)^{12t}$
b) $\approx 11,209.06$ dollars
c) $t \approx 12.6\ years$

22) a) $S = 6000\left(1 + \frac{0.072}{4}\right)^{4t}$
b) $\approx$ 8,572.49 dollars
c) $t \approx 15.4\ years$

23) a) $degree\ n = 3; leading\ coefficient\ a = 3$
b) $x = -6, -2, 2$
c) $f(x) \to -\infty\ as\ x \to -\infty,\ f(x) \to \infty\ as\ x \to \infty$
d) 2 turning points

24) a) $degree\ n = 3; leading\ coefficient\ a = 1$
b) $x = 4, -3, 3$
c) $f(x) \to \infty\ as\ x \to -\infty, f(x) \to -\infty\ as\ x \to \infty$.
d) 2 turning points

25) a) $x = -1, 0, 1$
b) $x = 0, 7, 8$
c) $x = 0, \frac{-5}{2}, 4$
d) $x = 0, 1, 2$

26) a) Domain: $(-\infty, 1) \cup (1, \infty)$; V.A.: $x = 1$; H.A.: $y = 1$
b) Domain: $(-\infty, \frac{-1}{2}) \cup (\frac{-1}{2}, \infty)$; V.A.: $x = \frac{-1}{2}$; H.A.: $y = \frac{-5}{2}$
c) Domain: $(-\infty, -5) \cup (-5, -2) \cup (-2, \infty)$; V.A.: $x = -2$, $(Hole\ at\ x = -5)$; H.A.: $y = 0$

27) a) 3; b) (−7,0), (−2,0), (2,0); c) $a < 0$; d) The minimum degree of f is 4.

28) a) 2; b) (−3,0); c) $a > 0$; d) The minimum degree of f is 3.